Confining and deconfining backgrounds in type IIB supegravity

Marcelo Oyarzo[1] in collaboration with Carlos Nuñez[2*] and Ricardo Stuardo[2#]

[1]Departamento de Física, Universidad de Concepción Casilla, 160-C, Concepción, Chile.

[2]Department of Physics, Swansea University, Swansea SA2 8PP, United Kingdom

[1]moyarzo2016@udec.cl *c.nunez@swansea.ac.uk #ricardostuardotroncoso@gmail.com

Introduction

Kaluza-Klein compactifications of the low energy limit of superstring theory lead to several gravitational models in lower dimension with different number of supersymmetries. The Freedman-Schwarz model [1] belongs to this class, which is obtain by compactifying on $S^3 \times S^3$ the NS sector. It gives the $\mathcal{N} = 4$ gauged supegravity in 4-dimensions. The bosonic field content of the theory is a metric, two $SU(2)$ gauge fields A and B, one dilaton Φ_{4d} and one axion field. It was found in [2] a new family of solution regarding black holes and solitons which we will analyse from the 10d perspective. The lift to Type IIB supergravity of the Freedman-Schwarz model was given in [3]

$$ds^2 = e^{2\Phi_{4d}} ds^2_{4d} + 2\left(\Theta^i_A\right)^2 + 2\left(\Theta^i_B\right)^2 ,$$
$$\Theta^i_A = A^i - \frac{1}{e_A}\hat{\omega}^i , \quad \Theta^i_B = B^i - \frac{1}{e_B}\tilde{\omega}^i ,$$

with $\hat{\omega}^i$ and $\tilde{\omega}^i$ the Maurer-Cartan forms for $su(2)$. The background is supported by the dilaton in 10D and the NS-NS 3-form

$$\phi = \Phi_{4d} ,$$
$$H_3 = 2\left(-F^i_A \wedge \Theta^i_A - F^i_B \wedge \Theta^i_B + e_A \Theta^1_A \wedge \Theta^2_A \wedge \Theta^3_A + e_B \Theta^1_B \wedge \Theta^2_B \wedge \Theta^3_B\right) .$$

This is a work in progress, further details will be given in the future.

Type IIB Supegravity and S-duality

We will work in Type IIB supergravity with only the 3-form field strength F_3 different than zero and the following normalization

$$S_{IIB} = \frac{1}{2\kappa^2_{10}} \int d^{10}x \sqrt{-g} e^{-2\phi} \left(R + 4\partial_\mu \phi \partial^\mu \phi - \frac{1}{2}|H_3|^2 - \frac{1}{2} e^{2\phi} |F_3|^2 + \dots \right) ,$$

Moving to the string frame, a solution of the Freedman-Schwarz model is also a solution of type IIB supergravity by using the up lift mentioned in the introduction. These solutions are in the NS sector, but due to the fact that type IIB string theory is S-self dual, we can use it to map the NS5-brane to D5-brane in our case. In order to increase the number flat directions where the dual theory is formulated, we consider the limit $e_B \to 0$ and $Q_B \to 0$ which corresponds to flattening one of the S^3.

The soliton solution $\mathbb{R}^3 \times S^3$

Performing the S-duality to the soliton found in [2] after the up-lift we get the following configuration with $H_3 = 0$ in the string frame

$$ds^2 = r\left\{ dx^2_{01234} + \frac{4dr^2}{r^2 f_s(r)} + f_s(r)\, d\varphi^2 + \frac{2}{e^2_A}\left[\hat{\omega}^2_1 + \hat{\omega}^2_2 + \left(\hat{\omega}_3 - \frac{e_A Q_A}{r^2} d\varphi\right)^2\right]\right\}$$
$$F_3 = -\frac{4}{r^3}\frac{Q_A}{e_A} dr \wedge d\varphi \wedge \hat{\omega}_3 + \frac{2}{e^2_A}\hat{\omega}_1 \wedge \hat{\omega}_2 \wedge \left(\frac{e_A Q_A d\varphi}{r^2} - \hat{\omega}_3\right)$$
$$\phi = \log r .$$

where $f_s = \frac{e^2_A}{2r^4}(r^2 - r^2_+)(r^2 - r^2_-)$ with $r^2_- = r^2_+ - m e^{-2}_A$. One of the coordinate is identified $\varphi \sim \varphi + 8\pi/r_0 f'_s(r_+)$, $t, x \in \mathbb{R}$ and $r \in [r_+, \infty[$. It has 4 independent killing spinors when $m = 0$.

Meron black hole $\mathbb{R}^3 \times S^3$

The single meron black hole found in [2] has the following 10D metric and gauge field, that we should replace in the equations in the introduction, are

$$ds^2_{10d} = r\left(\frac{\sqrt{2}}{e_A}\left(-f(r)dt^2 + d\Omega^2_2 + dy^2_{123}\right) + \frac{\sqrt{2}}{e_A}\frac{dr^2}{r^2 f(r)} + \sqrt{2}\, e_A \left(\Theta^i_A\right)^2\right)$$
$$F_3 = 2\left(-F^i_A \wedge \Theta^i_A + \frac{e_A}{3!}\epsilon_{ijk}\Theta^i_A \wedge \Theta^j_A \wedge \Theta^k_A\right) , \quad \phi = \ln\left(\frac{e_A r}{\sqrt{2}}\right) ,$$

where $U = \cos\varphi \sin\theta \alpha_1 + \sin\varphi \sin\theta \alpha_2 + \cos\theta \alpha_3$ and the metric function is $f(r) = \frac{1}{2} - \frac{m}{r^2}$. This configuration contains an S^2 bundled on S^3.

Wilson Loop

Following [4] in a generic background of the form

$$ds^2 = -g_{tt}dt^2 + g_{xx}d\vec{x}^2 + g_{rr}dr^2 + g_{ij}d\theta^i d\theta^j ,$$

where g_{tt}, g_{xx}, g_{rr} depend only on the radial coordinate r. The equations for the string moving in this background coming from the Nambu-Goto action reduces to

$$\frac{dr}{d\sigma} = \pm\frac{dx}{d\sigma}\frac{F(r)}{F(r_0)G(r)}\sqrt{F(r)^2 - F(r_0)^2} ,$$

$F(r)^2 = g_{tt}g_{xx}$, $G(r)^2 = g_{tt}g_{rr}$. This represents an open string whose end points are at $r \to \infty$, then we add a D-brane at vary large radial distance. We can compute the separation between the two ends of the string in the D-brane, which can be thought as the separation between a quark-antiquark pair living in the D-brane, as well as the energy of the pair of quarks. These quantities are given by

$$L_{QQ}(r_0) = 2\int_{r_0}^{+\infty} dr \frac{F(r_0)G(r)}{F(r)\sqrt{F(r)^2 - F(r_0)^2}} , \qquad \frac{dE_{QQ}}{dL_{QQ}} = F(r_0)$$
$$E_{QQ}(r_0) = F(r_0) L_{QQ}(r_0) + 2\int_{r_0}^{+\infty} dz \frac{G(z)}{F(z)}\sqrt{F(z)^2 - F(r_0)^2} - 2\int_{r_+}^{+\infty} dz\, G(z) ,$$

where r_0 is the turn around point for the string in the bulk. The differential relation between the energy and the length given above can be derived analytically, to use it we must invert the relation between r_0 and L_{QQ}.

Confining background: Soliton

This background represents a $D5$-brane on $S^1 \times S^3$. The integrals above can be computed analytically. They are given in terms of Elliptic functions

$$L_{QQ} = \frac{4r_0}{e_A\sqrt{r^2_+ - r^2_-}}\left[i\mathbf{K}\left(1 - \frac{r^2_0 - r^2_-}{r^2_+ - r^2_-}\right) + \mathbf{K}\left(\frac{r^2_0 - r^2_-}{r^2_+ - r^2_-}\right)\right] .$$

for $r_0 \in]r_+, \infty[$ we have $L_{QQ} \in]\frac{2\pi}{e_A}, \infty[$. The length goes to infinity when the turn around point goes to the end of the space at r_+. In this limit we can write $r_0 = r_0(L_{QQ})$ and using the differential relation we find the energy as a function of the length

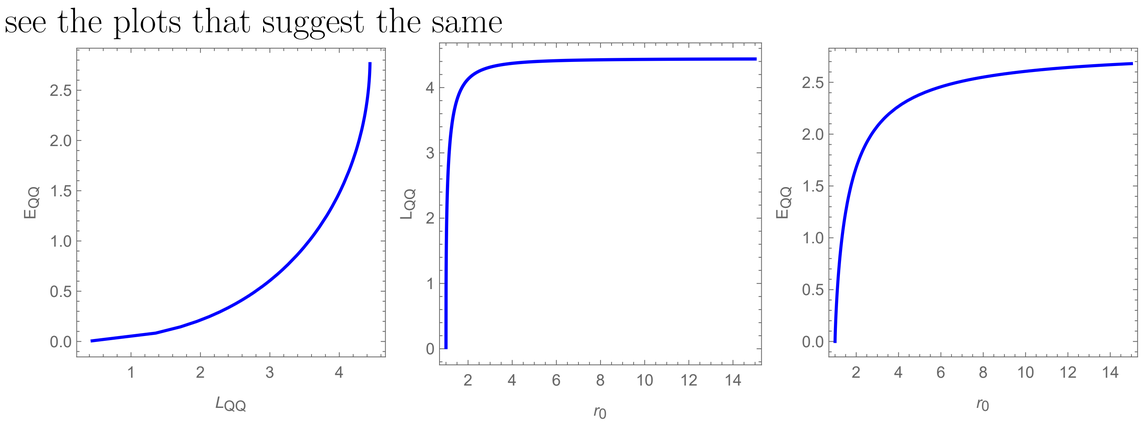
$$E_{QQ} = r_+ L_{QQ} + \mathcal{O}\left(e^{-\frac{2}{r_+}e_A L_{QQ}\sqrt{r^2_+ - r^2_-}}\right)$$

Deconfining background: Meron black hole

In the case of the black hole the integrals also can be done analytically, but the resulat are very different. The separation between the quarks is given by

$$L_{QQ}(r_0) = 2\sqrt{2}\sqrt{\frac{r^2_0}{r^2_+} - 1}\left[i\mathbf{K}\left(1 - \frac{r^2_0}{r^2_+}\right) + \mathbf{K}\left(\frac{r^2_0}{r^2_+}\right)\right] .$$

In this case the separation is bounded between two value $L_{QQ} = [0, \sqrt{2}\pi[$ then, there is no way to obtain confinement in this background. Also we can see the plots that suggest the same



References

[1] D. Z. Freedman and J. H. Schwarz, Nucl. Phys. B **137** (1978), 333-339

[2] F. Canfora, J. Oliva and M. Oyarzo, JHEP **02** (2022), 057.

[3] A. H. Chamseddine and M. S. Volkov, Phys. Rev. D **57** (1998), 6242-6254

[4] C. Nunez, M. Piai and A. Rago, Phys. Rev. D **81** (2010), 086001

Acknowledgement This work is partially funded by Beca ANID de Doctorado 21222264 and FONDECYT grants 1221504.

Nordgrav@Iquique2023