(1+1) confining QFT in type IIB string theory

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In this work we study a background in type IIB supergravity describing two intersecting stacks of D5-branes [1]. The intersection of the stacks is a 3-dimensional space with a contractible cycle, then we expect a QFT in (1+1) dimensions at weak coupling.

Supergravity background

The explicit form of the metric, the 2-form potential of the Ramond 3-form and the dilaton in coordinates $(t, x, \varphi, r, \theta_A, \phi_A, \psi_A, \theta_B, \phi_B, \psi_B)$ in string frame is given by

$$\begin{aligned} ds^2_{st} = r\Big\{ & -dt^2 + dx^2 + f_s(r)\, d\varphi^2 + \frac{4dr^2}{r^2 f_s(r)} \\ & + \frac{2}{e_A^2}\left[\hat{\omega}_1^2 + \hat{\omega}_2^2 + (\hat{\omega}_3 - e_A Q_A \zeta(r)\, d\varphi)^2\right] \\ & + \frac{2}{e_B^2}\left[\tilde{\omega}_1^2 + \tilde{\omega}_2^2 + (\tilde{\omega}_3 - e_B Q_B \zeta(r)\, d\varphi)^2\right]\Big\} \ , \\ C_2 = {} & \psi_A\left(\frac{2Q_A}{e_A}\zeta'(r)\, dr \wedge d\varphi - \frac{2}{e_A^2}\sin\theta_A d\theta_A \wedge d\phi_A\right) \\ & + \psi_B\left(\frac{2Q_B}{e_B}\zeta'(r)\, dr \wedge d\varphi - \frac{2}{e_B^2}\sin\theta_B d\theta_B \wedge d\phi_B\right) \\ & + 2\zeta(r)\, d\varphi \wedge \left(\frac{Q_A}{e_A}\cos\theta_A d\phi_A + \frac{Q_B}{e_B}\cos\theta_B d\phi_B\right) \\ F_3 = {} & dC_2 \ , \qquad \Phi = \log r \ . \end{aligned}$$

where $\hat{\omega}_i(\theta_A, \phi_A, \psi_A)$, $\tilde{\omega}_i(\theta_B, \phi_B, \psi_B)$ are the left invariant forms of $SU(2)$. The function $f(r)$ and $\zeta(r)$ have one zero at r_+ and are given by

$$f(r) = \frac{e_A^2 + e_B^2}{2} - \frac{m}{r^2} - \frac{2\left(Q_A^2 + Q_B^2\right)}{r^4} \ , \quad \zeta(r) = \frac{1}{r^2} - \frac{1}{r_+^2} \ .$$

Here r_+ is the only real zero of $f_s(r)$ and depends on the parameters (m, e_A, Q_A, e_B, Q_B). This background has a fibration between the 3-spheres and the contractible cycle φ with fixed periodicity. It preserves four supercharges when $m = 0$ and $e_A Q_B = e_B Q_A$.

The $D5$ charges associated to the Ramond field F_3 can be computed in the three cycles $\mathcal{M}_{A,B} = (\psi_{A,B}, \theta_{A,B}, \phi_{A,B})$ giving

$$N_{A,B} = \frac{1}{(2\pi)^2}\int_{\mathcal{M}_{A,B}} F_3 \quad \Longrightarrow \quad N_A = \frac{8}{e_B^2} \ , \quad N_B = \frac{8}{e_A^2} \ ,$$

implying a quantization conditions for the parameters (e_A, e_B). This result implies the presence of two stacks of D5 branes with N_A and N_B branes in each stack respectively.

Comments about the background:

- For large values of r the dilaton becomes large and the type IIB system is better described by an S-dual system in terms two stacks of intersecting NS five branes, which is dual to two Little String Theory (LST) intersecting on (t, x, φ). This implies that the QFT is UV completed by a non-local theory.
- At weak coupling the D5-D5 system preserves chiral supercharges and following [2] the spectrum in the intersection contains chiral fermions which give rise to gauge anomalies, that is cured by inflow from the bulk of the D5 branes.
- At strong coupling in the filed theory the system preserves $SO(2,1)$ with $SO(4) \times SO(4)$ R-symmetry and sixteen supercharges [3]. The 3-dimensional field theory has gauge groups with Chern-Simons terms $SU(N_A)_{N_B} \times SU(N_B)_{N_A}$.

1 Field theory

Let us compute some quantities in the field theory:

- **Yang-Mills coupling:** Considering a probe D5 brane along the coordinates $(t, x, \varphi, \theta_A, \phi_A, \psi_A)$. From Dirac-Born-Infeld of this be we identifying the Yang-Mills coupling

$$\frac{1}{g^2_{YM,A}} = 2\pi^4 T_{D5} L_\varphi \sqrt{\left(1 - \frac{r_+^2}{r^2}\right)\left(\frac{r_+^2}{r^2} + 1 + \frac{2e_B^2}{e_A^2}\right)} \ .$$

At low energy the gauge coupling grows very large and for high energies it asymptotes to a constant value.

- **Θ-angle for one of the gauge groups:** Let us consider a $D3$ probe extended along $[t, x, \theta, \phi_A]$ with an electric field F_{tx} turned on. The Wess-Zumino terms for this probe is

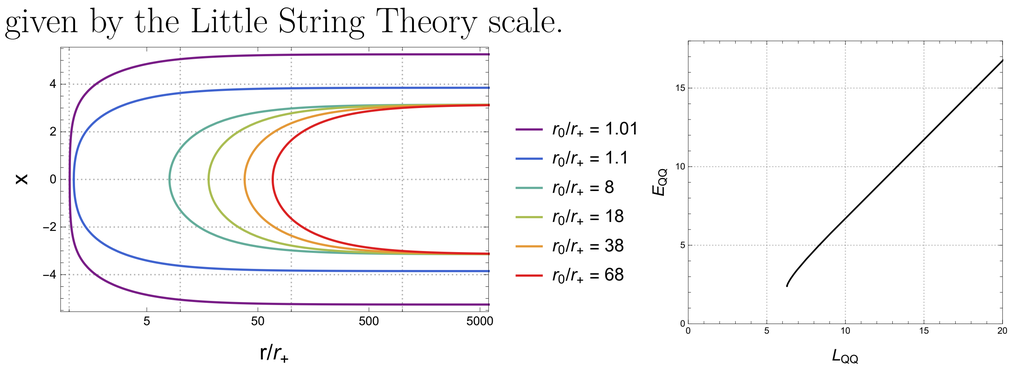
$$S_{WZ,D3} = \frac{16\pi^2 T_3}{e_A^2}\psi_A \int F_{tx} dt \wedge dx \quad \Longrightarrow \quad \Theta_A = \frac{\psi_A N_A}{2} \ .$$

where we have identified the pre-factor of the integral with $\Theta_A/\left(4\pi^2\right)$, the same happens changing $A \to B$. The periodic identification $\Theta_{A,B} \sim \Theta_{A,B} + 2k\pi$ implies that $\psi_{A,B}$ gets quantized to the valued $\Delta\psi_{A,B} = \frac{4k\pi}{N_{A,B}}$ with $k = 0, 1, \ldots N_{A,B} - 1$.

- **R-Symmetry breaking pattern:** To better understand the breaking of the isometries $\partial_{\psi_{A,B}}$ we gauge these isometry by replacing $d\psi_{A,B} \to d\psi_{A,B} + A_{A,B}$ and $\psi_{A,B} \to \psi_{A,B} + \epsilon_{A,B}$. Replacing into the supergravity action we find that the perturbations acquires a mass term.

In what follows we put a D-brane at $r \to \infty$ on this background and a macroscopic open string (or a section of a higher dimensional object) attached to it, which dynamics is governed by the Nambu-Goto (effective) action. The profiles are U-shaped with turn-around point at $r_0 > r_+$.

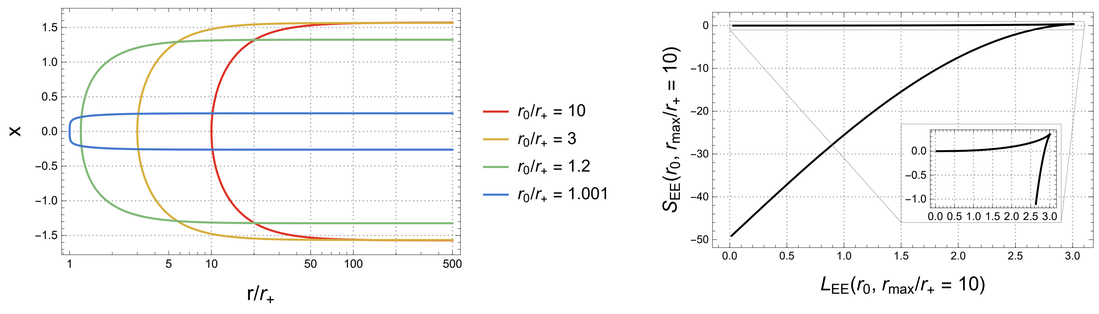
- **Maldacena-Wilson loop:** The end points are interpreted as a pair of quark-anti-quark with separation L_{QQ} and energy E_{QQ}. We find that (i) the concavity of the curve $E_{QQ}(L_{QQ})$ is "downwards" indicating the stability of the string configuration. (ii) For large separation L_{QQ} the energy grows linearly, which is a signal of confinement. (iii) For short separation L_{QQ} the strings barely explores the bulk and it has a minimum separation given by the Little String Theory scale.



- **Entanglement Entropy:** The Entanglement entropy between two regions for a field theory with a string dual can be computed minimizing the following functional for an 8 surface Σ_8

$$S_{EE}[\Sigma_8] = \frac{1}{4G_N}\int_{\Sigma_8} d^8\sigma \sqrt{e^{-4\Phi} g_{\Sigma_8}} \ .$$

with proper boundary conditions. We focus on the case of a strip of length L_{EE} as one of the entangled regions and the 8 surfaces is along $[x, \varphi, \theta_A, \phi_A, \psi_A, \theta_B, \phi_B, \psi_B]$ and $r(x)$. It was suggested in [4]-[5] that a signal of confinement is the presence of a phase transition in the EE but it is absent for non-local UV completion. We find that (i) The curve $S_{EE}(L_{EE})$ has the "upwards"concavity indicating the instability of the configuration. (ii) Taking out the Little String Theory dynamics in a proper way, including a cutoff, the plot $S_{EE}(L_{EE})$ shows the correct concavity and the presence of a phase transition to the disconnected configuration.



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Acknowledgement This work is partially funded by Beca ANID de Doctorado 21222264

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