Stability of Black Holes in the STU Model of $D = 4, \mathcal{N} = 8$ Gauged Supergravity

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Outline

- Motivations and preliminary observations
- STU model and different truncations
- Example in Einstein-Maxwell $\subset$ STU model

- Electrically charged black holes in the STU model
  - Result: Instability of low temperature electrically charged black holes
- Magnetically charged black holes in the STU model
  - Proposal: Energy functional that makes BPS magnetic black holes stable

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- Final Remarks

Phase space

The phase space of a theory is defined as the set of configurations that satisfy the equation of motion together with some prescribed *boundary conditions.*

- ▷ Find gauge transformations $\lambda(x)$ that preserve the boundary conditions.
- ▷ Compute the charges $Q[\lambda] = \int_{\partial\Sigma} \boldsymbol{q}(\lambda, \text{fields})$.
- ▷ $\{Q, Q\}_{\text{DB}}$ algebra of the symmetry group of your theory

Bound in supersymmetric theories

Supersymmetric theories have a symmetry generated by fermionic parameters ϵ^A

$$Q_{A\alpha} = \int_{\Sigma_{D-1}} \star S_{A\alpha}$$

$$\{Q_{A\alpha}, Q_B{}^{\beta}\} = (\Gamma^{\mu}{}_{AB})_{\alpha}{}^{\beta} P_{\mu} + \begin{matrix}\text{central}\\ \text{charges}\end{matrix}$$

The superalgebra implies a bound on the energy [Witten-Olive 78]

$$P_0 \geq \begin{matrix}\text{central}\\ \text{charges}\end{matrix}$$

The equality sets the vacuum state in that sector of the theory.

Bound in supersymmetric theories

The gravitino Ψ_μ^A is the superpartner of the metric $g_{\mu\nu}$.

$$\epsilon^A \to \epsilon^A(x) \qquad \Longrightarrow \qquad Q_{A\alpha} = \int_{\Sigma_{D-1}} \mathrm{d}\mathfrak{q}_{A\alpha} = \int_{\partial\Sigma_{D-1}} \mathfrak{q}_{A\alpha}$$

Integral on a codim-2 surface.

One can compute the Dirac bracket [Hristov,Toldo,Vandoren 2011][Hristov 2011]

$$\{\mathcal{Q}, \mathcal{Q}\} = 2 \int_{\partial\Sigma_{D-1}} \bar{\epsilon}_A \gamma \wedge \mathfrak{D}^A(\epsilon) - \bar{\epsilon}^A \gamma \wedge \mathfrak{D}_A(\epsilon)$$

$$\mathcal{Q} \equiv \epsilon^{A\alpha} Q_{A\alpha}$$
$$\delta\Psi_\mu^A \mathrm{d}x^\mu \equiv \mathfrak{D}^A(\epsilon)$$

Observation 1: BPS states in supergravity

The supersymmetry transformations are (schematically)

$\frac{1}{4}\omega_{ab}\gamma^{ab} - \frac{1}{4}F_{ab}\gamma^{ab}\gamma_c e^c + \frac{1}{L}(\dots)$

$$\delta\Psi^A_\mu \mathrm{d}x^\mu = \mathrm{d}\epsilon^A + \mathbf{W}^A{}_B\epsilon^B = 0$$
$$\delta\lambda^{iA} = \mathbf{M}^A{}_B\epsilon^B = 0$$

BPS configuration $\equiv \exists \epsilon^A \neq 0$ that satisfy the above equations.

$$P_0(\text{ parameters })|_{\text{BPS}} = \begin{matrix}\text{central}\\ \text{charges}\end{matrix}$$

Problem: In supergravities there are BPS states that are naked singularities

Observation 2

Black holes are thermodynamic *objects*

$$S_{\mathrm{BH}} = \frac{k_B c^3 A}{4G\hbar}$$
$$T_{\mathrm{H}} = \frac{\kappa}{2\pi}$$

[Bekenstein, 72, 73, 74][Hawking, 74, 75]

The question regarding thermodynamic stability is well posed in this context.

Configurations on AdS exhibit a rich phase diagram [Hawking,Page 83] [Chamblin,Emparan,Johnson,Myers 99][Gubser,Mitra 2000][Donos,Gountlett,Pantelidou 2011×2]...

Goal #1:

Revisit the BPS limit of non-BPS static black holes

Goal #2:

Study the thermodynamic stability of (near-)extremal black holes

Gauged STU model $D = 4, \mathcal{N} = 2$

Construction of the STU model

The field content of the $\mathcal{N}=8$ SO(8)-gauged supergravity

spin	field	Rep of SU(8)	#
2	$g_{\mu\nu}$	$-$	1
$3/2$	$\Psi_\mu^{\mathbb{A}}$	$\mathbf{8}$	8
1	$A_\mu^{\mathbb{AB}}$	$\mathbf{8}\wedge\mathbf{8}$	28
$1/2$	$\lambda^{\mathbb{ABC}}$	$\mathbf{8}\wedge\mathbf{8}\wedge\mathbf{8}$	56
0	$\phi^{\mathbb{ABCD}}$	$\mathbf{8}\wedge\mathbf{8}\wedge\mathbf{8}\wedge\mathbf{8}$	70

The complex scalars satisfy the reality condition and the scalar manifold is

$$\phi_{\mathbb{ABCD}} = \frac{1}{4!}\varepsilon_{\mathbb{ABCDEFGH}}(\phi_{\mathbb{EFGH}})^*\,, \qquad \mathscr{M}_{\text{scal}} = \frac{E_{7(7)}}{\text{SU}(8)}\,.$$

Construction of the STU model

Retain the singles under $H_0 = SO(2)_1 \times SO(2)_2 \times SO(2)_3 \subset \mathrm{SO}(8) \subset \mathrm{SU}(8)$

[Duff,Liu 98]

$$\begin{pmatrix} SO(2)_0 & 0 & 0 & 0 \\ 0 & SO(2)_1 & 0 & 0 \\ 0 & 0 & SO(2)_2 & 0 \\ 0 & 0 & 0 & SO(2)_3 \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \\ \psi_5 \\ \psi_6 \\ \psi_7 \\ \psi_8 \end{pmatrix}$$

$\mathcal{N}=2$ gauged STU model

The bosonic action principle is

$$\frac{1}{2\kappa}\int \mathrm{d}^4x\sqrt{-g}\left(\frac{R}{2}-\mathbf{g}_{i\bar{j}}\partial_\mu z^i\partial^\mu\bar{z}^{\bar{j}}-V(z,\bar{z})+\frac{1}{4}\mathcal{I}_{\Lambda\Sigma}F^\Lambda_{\mu\nu}F^{\Sigma\mu\nu}+\frac{1}{8e}\mathcal{R}_{\Lambda\Sigma}F^\Lambda_{\mu\nu}F^\Sigma_{\rho\sigma}\varepsilon^{\mu\nu\rho\sigma}\right)$$

Bosonic fields:

$g_{\mu\nu}$ metric

z^i complex scalar $i=1,2,3.$ Coordinates of a *Special Kähler* manifold.

A^Λ gauge vectors $\Lambda=1,2,3,4.$ Duality group is $\mathrm{Sp}(8,\mathbb{R})$.

Fayet-Iliopoulos (FI) terms: $\theta_M=\frac{1}{\sqrt{2}L}(1,1,1,1,0,0,0,0)$

SUSY transformations

The supersymmetry transformations for the fermions are

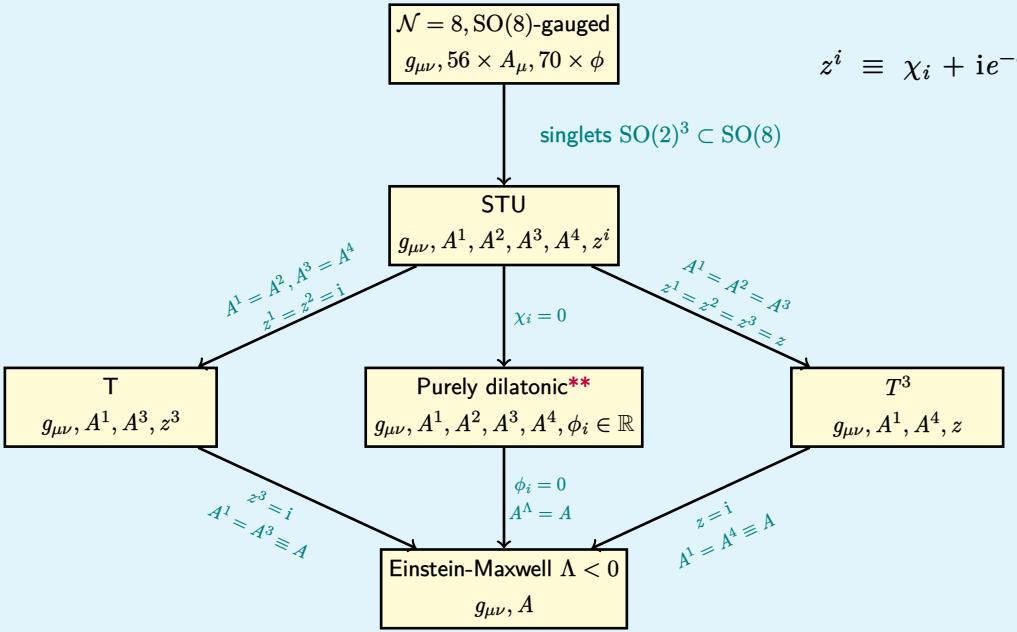
$$\delta\Psi^A_\mu \mathrm{d}x^\mu = D\epsilon^A + \frac{1}{4} L^T \mathcal{I} \not{F} \gamma \varepsilon^{AB} \epsilon_B + \frac{1}{2} \mathcal{W} \gamma \delta^{AB} \epsilon_B \,,$$

$$D\epsilon^A = \mathrm{d}\epsilon^A + \frac{1}{4}\omega_{ab}\gamma^{ab}\epsilon^A + \frac{1}{2} A^M \theta_M \varepsilon^{AB} \delta_{BC} \epsilon^C + \frac{\mathrm{i}}{2} \mathbb{Q}_\mu \mathrm{d}x^\mu \epsilon^A \,,$$

$$\delta\lambda^{iA} = -\not{\partial} z^i \epsilon^A + \frac{1}{2} g^{i\bar{j}} \overline{f}^T_{\bar{j}} \mathcal{I} \not{F} \varepsilon^{AB} \epsilon_B + g^{i\bar{j}} \overline{\mathcal{U}}^M_{\bar{j}} \theta_M \delta^{AB} \epsilon_B$$

the spinorial parameters are chiral spinors ϵ^A, ϵ_A

$$\gamma_5 \epsilon_A = \epsilon_A \qquad \gamma_5 \epsilon^A = -\epsilon^A \qquad (\epsilon_A)^* = \epsilon^A$$



$$z^i \equiv \chi_i + \mathrm{i}e^{-\phi_i}$$

**consistent plus constraint

Example: Reissner-Nordström

Dyonic *spherical* Reissner-Nordström-AdS (M, Q, P)

$$\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\varphi^2)\,, \qquad f(r) = \frac{r^2}{L^2} + 1 - \frac{2M}{r} + \frac{Q^2+P^2}{r^2}$$

$$A_\mu \mathrm{d}x^\mu = \frac{Q}{\sqrt{2}r}\mathrm{d}t + \frac{P}{\sqrt{2}}\cos\theta \mathrm{d}\varphi$$

BPS equations are satisfied in *two different ways* [Romans 92]

Option 1 : $M = |Q|\,, \qquad P = 0$

Electric 1/2 BPS

Naked singularity

Option 2 : $M = 0\,, \qquad P = \pm\frac{L}{\sqrt{8}}$

Dyonic 1/4 BPS

Naked singularity

Two different boundary conditions

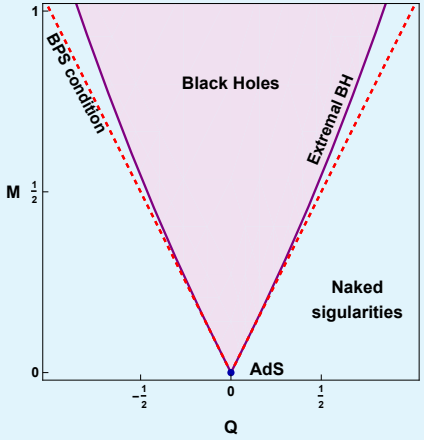
That allow a BPS limit

Option 1: Electric boundary conditions

$$\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\varphi^2) \qquad f(r) = \frac{r^2}{L^2} + 1 - \frac{2M}{r} + \frac{Q^2}{r^2}$$

$$= \mathrm{AdS}_4 + o(r^{-1})$$

$$A = \left(\frac{Q}{\sqrt{2}r} + \mu\right)\mathrm{d}t = o(r^0)$$



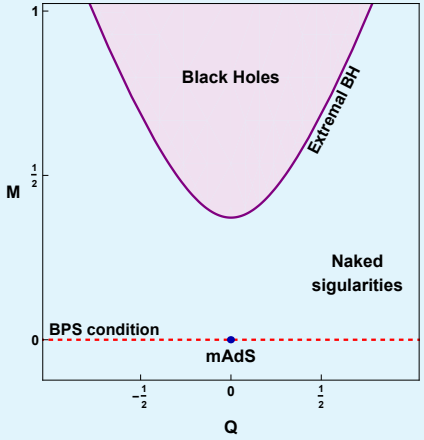
Option 2: Magnetic boundary conditions

$$\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\varphi^2) \qquad f(r) = \frac{r^2}{L^2} + 1 - \frac{2M}{r} + \frac{Q^2 + L^2/8}{r^2}$$

$$= \mathrm{AdS}_4 + o(r^{-1})$$

$$A = \frac{Q}{\sqrt{2}r}\mathrm{d}t + \frac{L}{4}\cos\theta \mathrm{d}\varphi$$

$$= \frac{L}{4}\cos\theta \mathrm{d}\varphi + o(r^{-1})$$



How to generalize spherical Reissner-Nordström-AdS (M, Q, P)?

$\triangleright$ Adding rotation (M, Q, P, J) [Kostelecky,Perry 94] Rotating BPS black hole

$\triangleright$ Horizon with different topology [Caldarelli,Klemm 98] Hyperbolic BPS black holes

$\triangleright$ BPS Accelerating, rotating and dyonically charged$\subset$Plebański-Demiański family

$\hookrightarrow$ $11D$ SUGRA regular configuration [Ferrero,Gauntlett,Pérez, Martelli, Sparks 2020]

$\triangleright$ etc.

New solutions adding scalars?

$\triangleright$ 4 charges electric or magnetic dilatonic [Duff,Liu 98][Cvetic,Duff,Hoxha, et.al.98]

$\triangleright$ 4 charges *magnetic BPS spherical black holes* with dilatons [Cacciatori,Klemm 2010]

$\triangleright$ 8 charges dilatonic-axionic $(M, Q_\Lambda, P^\Lambda)$ static black holes [Compere,Chow 2013]

$\triangleright$ etc.

Black hole solution in the dilatonic sector of the STU model

- Electric black holes
- Magnetic black holes

...Why separated?

Electric hairy black hole

The black hole configuration [Duff,Liu 98]

$$\mathrm{d}s^2 = -\frac{f(r)}{H(r)^{1/2}}\mathrm{d}t^2 + \frac{H(r)^{1/2}}{f(r)}\mathrm{d}r^2 + r^2 H(r)^{1/2}\mathrm{d}\Sigma_{2,k}^2$$

$$f(r) = \frac{r^2}{L^2}H(r) + k - \frac{m}{r} - \frac{\mathtt{q}}{r^2}\,, \qquad H = \prod_{\Lambda} H_\Lambda\,, \qquad H_\Lambda = 1 + \frac{q_\Lambda}{r}$$

and

$$\phi_i = \frac{1}{2}\log\left(\frac{H_j H_k}{H_i H_4}\right) \qquad \begin{cases} j = i+1 \in \{1,2,3\} \\ k = i+2 \in \{1,2,3\} \end{cases}$$

$$A^\Lambda = \frac{1}{\sqrt{2}}\left(\frac{Q_\Lambda}{r H_\Lambda} + \Phi_\Lambda\right)\mathrm{d}t\,,$$

$$Q_\Lambda^2 = q_\Lambda^2 k + q_\Lambda m - \mathtt{q}\,.$$

$\mathtt{q}$ was introduced shifting r.

Electric hairy black holes $\to$ BPS ☹

The BPS limit of the hairy black holes is

$$k\mathrm{q} = -\frac{m^2}{4}$$

1/2 BPS *Naked singularities* for spherical and planar configurations.

▷ For fixed k the BPS condition *does not restrict* your boundary conditions.

▷ Hyperbolic case leads imaginary vector fields.

Instanton?

Killing spinor electric backgrounds

Planar ($k = 0$) Killing spinor:

$$\psi^A_{\rm pl}(r) = \frac{f_{\rm pl}^{1/4}(r)}{H^{1/8}(r)}(\alpha_{\rm pl}(r) - \beta_{\rm pl}(r)\gamma_1)(\delta_{AB} + \varepsilon_{AB}\gamma_0)\psi_0^B\,,$$

Spherical ($k = 1$) Killing spinor:

$$\psi^A_{\rm sp}(t, r, x, y) = \frac{f_{\rm sp}^{1/4}(r)}{H^{1/8}(r)} e^{\frac{it}{2L}} e^{\frac{i}{2}\gamma_{012}\arccos x} e^{-\frac{1}{2}y\gamma_{23}}(\alpha_{\rm sp}(r) + \beta_{\rm sp}(r)\gamma_1)(\delta_{AB} + \varepsilon_{AB}\gamma_0)\psi_0^B\,,$$

Now, let us consider only the planar case $k = 0$

(Still in the electric case)

Equation of state

In the planar black holes case $k = 0$

$$m = \frac{1}{A^{1/2}} \prod_{\Lambda=1}^{4} \left(\frac{A^2}{L^2} + Q_\Lambda^2 \right)^{1/4}$$

In the electric case m is identified with the energy E

$$\delta E = 8\pi T_H \delta S + \sum_\Lambda \Phi_\Lambda \delta Q_\Lambda$$

The stability of the system can be studied the Hessian matrix $\mathcal{H}_{ab} \geq 0$.

Proposition

All the electric planar nearly-extremal black holes are thermodynamically unstable.

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Proof.

As m is the energy:

$$E(Q_\Lambda, A) = \frac{1}{A^{1/2}} \prod_\Lambda \left(\frac{A^2}{L^2} + Q_\Lambda^2 \right)^{1/4}$$

We compute the 5×5 Hessian matrix:

$$\mathcal{H}_{ab} = \frac{\partial^2 E}{\partial l^a \partial l^b}\,, \qquad l^a = (A/L, Q_\Lambda)$$

The determinant is simply written as:

$$\det \mathcal{H}_{ab} = \frac{\pi L^2}{4A^3}\, T_{\rm H} - \frac{1}{16 E^3 L^4} \sum_\Lambda Q_\Lambda^2\,.$$

Instability happen for

$$T_{\rm H} < \frac{A^3}{4\pi E^3 L^8} \sum_\Lambda Q_\Lambda^2$$

□

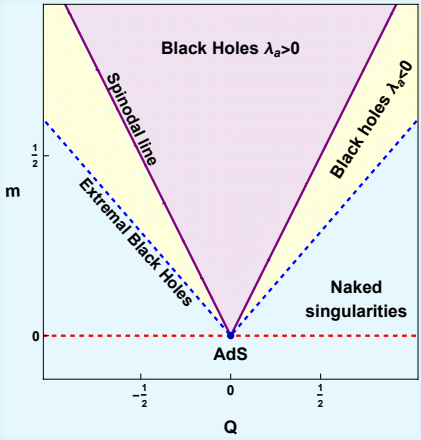
Electric: specialization

We can go to Einstein-Maxwell by setting $Q_\Lambda = Q$

$$\xi \equiv \frac{L^2 Q^2}{A^2}$$

1× $$\lambda_\pm = \frac{1}{8A^{1/2}}\left(5 + 3\xi \pm \sqrt{1 + 22\xi + 9\xi^2}\right) > 0$$

3× $$\lambda = \frac{1}{2A^{1/2}}\frac{1-\xi}{1+\xi}$$



Moving to the magnetic hairy black holes

Magnetic planar hairy black holes

The black hole configuration [Duff,Liu 98]

$$\mathrm{d}s^2 = -\frac{f(r)}{H(r)^{1/2}}\mathrm{d}t^2 + \frac{H(r)^{1/2}}{f(r)}\mathrm{d}r^2 + r^2 H(r)^{1/2}\mathrm{d}\Sigma_{2,k}^2$$

$$f(r) = \frac{r^2}{L^2}H(r) + k - \frac{m}{r} - \frac{\mathtt{q}}{r^2}\,, \qquad H = \prod_{\Lambda} H_\Lambda\,, \qquad H_\Lambda = 1 + \frac{q_\Lambda}{r}$$

and

$$\phi_i = -\frac{1}{2}\log\left(\frac{H_j H_k}{H_i H_4}\right) \qquad \begin{cases} j = i+1 \in \{1,2,3\} \\ k = i+2 \in \{1,2,3\} \end{cases}$$

$$A^\Lambda = \frac{P_\Lambda}{\sqrt{2}} x \mathrm{d}y\,,$$

$$P_\Lambda^2 = q_\Lambda^2 k + q_\Lambda m - \mathtt{q}\,. \qquad \mathtt{q} \text{ was introduced shifting r.}$$

BPS limit of magnetic hairy black holes

The BPS limit give you extremal black holes satisfying

$$\sum_{\Lambda} P_{\Lambda} = \sqrt{2}k \qquad =: \text{Topological twist condition}$$

$$m^2 \equiv m^2_{\text{BPS}} \equiv \frac{1}{\sqrt{8}L} \prod_{i=1}^{3} \left(\sum_{\Lambda} \Omega_{i\Lambda} P^2_{\Lambda} \right)^{1/2}, \qquad \Omega_{i\Lambda} = \begin{pmatrix} 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \end{pmatrix}$$

$$f(r) = \left(\frac{r}{L} - \frac{A(P_{\Lambda})}{r} + B(P_{\Lambda}) \right)^2 \qquad \text{Extremal black hole}$$

Killing spinor of the planar BPS black hole

From now on we consider planar configurations $k = 0$

The Killing spinor in that case is

$$\psi^A(r) = \frac{f(r)^{1/4}}{4H(r)^{1/8}}(1+\gamma_1)(\delta_{AB} - \varepsilon_{AB}\gamma_{23})\psi_0^B \,.$$

Preserves 2 supercharges: 1/4-BPS.

Tension: Planar magnetic case

In the general case it possible to find

$$m = \frac{1}{A^{1/2}} \prod_{\Lambda} \left(\frac{A^2}{L^2} + P_{\Lambda}^2 \right)^{1/4}$$

Tension: now extremal black holes can be BPS and they are not unstable.

It is not clear that m represents the energy, but yet the first law holds

$$\delta m = 8\pi T_H \delta S + \textstyle\sum_{\Lambda} \Phi_{\Lambda} \delta P_{\Lambda}$$

We can propose a energy of the form: $E = m(A, P_{\Lambda}) + \tilde{m}(P_{\Lambda})$

We purpose a way to resolve the tension

- ▷ Find boundary conditions that admit asymptotic Killing spinors.

- ▷ Compute the BPS bound coming from the superalgebra.

Asymptotic Killing spinor

When the magnetic black holes have asymptotic Killing spinors?

For the following boundary conditions

$$\delta\Psi_\mu^A = o(r^{-2}) \qquad \delta\lambda^{iA} = o(r^{-2})$$

The asymptotic Killing spinor exists when the background satisfies

$$\sum_\Lambda P_\Lambda = 0 \qquad \text{(Topological twist condition)}$$

$$\psi^A_{\pm,\text{asym}} = r^{1/2}\mathbb{P}_\pm\psi_0^A + o(r^{-1/2}) \qquad \mathbb{P}_\pm = \frac{1}{4}(1+\gamma_1)(\delta_{AB} \pm \varepsilon_{AB}\gamma_{23})\,.$$

The most general asymptotic Killing spinor is $\psi^A_{\text{asym}} = \psi^A_{+,\text{asym}} + \psi^A_{-,\text{asym}}$

Computing the BPS bound

The bound in our case

$$\{\mathcal{Q}_\pm, \mathcal{Q}_\pm\} = 2\int \overline{\psi}^A_{\pm,\text{asy}}\gamma_5\gamma \wedge \left(\delta_{AB}\mathrm{d} + \tfrac{1}{4}\omega_{ab}\gamma^{ab}\delta_{AB} + \tfrac{1}{4}L^T\mathcal{I}F_{ab}\gamma^{ab}\gamma\varepsilon_{AB} + \tfrac{1}{2}\gamma\mathcal{W}\delta_{AB}\right)\psi^B_{\pm,\text{asy}}$$

↖ Computed in the background with $\sum_\Lambda P_\Lambda = 0$

$$\{\mathcal{Q}_\pm, \mathcal{Q}_\pm\} = \mathrm{i}\mathbf{M}_\pm\overline{\psi}^A_0\gamma^0\mathbb{P}^\pm_{AB}\psi^B_0\int \mathrm{d}x\wedge\mathrm{d}y$$

BPS bound: $\quad \mathbf{M}_\pm \geq 0 \qquad \mathbf{M}_\pm \equiv \frac{L}{3m^3}(m^2 \pm m^2_{\text{BPS}})(2m^2 \pm m^2_{\text{BPS}})$

The non-trivial BPS bound implies

$$\mathbf{M}_- \geq 0 \implies m(A, P_\Lambda) - m_{\text{BPS}}(P_\Lambda) \geq 0$$

Proposal of resolution of the tension

We propose the energy functional to be

$$E = m(A, P_\Lambda) - m_{\mathrm{BPS}}(P_\Lambda)$$

where

$$m(A, P_\Lambda) = \frac{1}{A^{1/2}} \prod_\Lambda \left(\frac{A^2}{L^2} + P_\Lambda^2 \right)^{1/4} \qquad m_{\mathrm{BPS}}(P_\Lambda) = \frac{1}{8^{1/4} L^{1/2}} \prod_i \left(\sum_\Lambda \Omega_{i\Lambda} P_\Lambda^2 \right)^{1/4}$$

For this energy the first law still holds for certain chemical potential Φ_Λ and

Claim (?)

The energy E has semi-positive defined Hessian matrix for black holes satisfying the topological twist $\sum_\Lambda P_\Lambda = 0$.

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For this energy the first law still holds for certain chemical potential Φ_Λ and

Claim (?)

The energy E has semi-positive defined Hessian matrix for black holes satisfying the topological twist $\sum_\Lambda P_\Lambda = 0$.

Not true. We need to specialize even more.

Extremal black holes $\Longrightarrow$ BPS black holes?

Extremality condition

All the extremal black holes satisfy $T = \dfrac{\partial E}{\partial S} = 0$

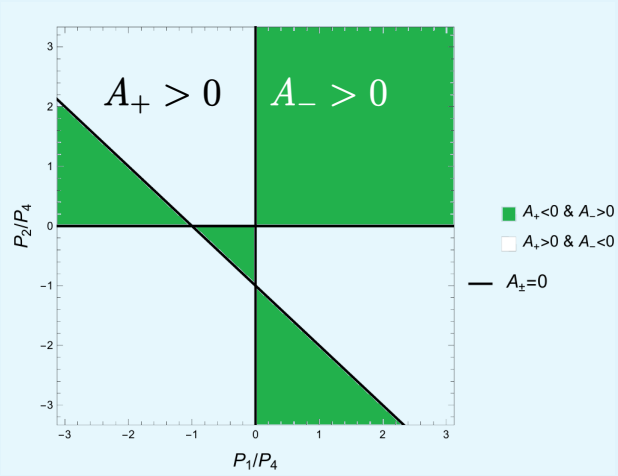
Imposing from the beginning that $\sum_\Lambda P_\Lambda = 0$

$$T = 0 \qquad \Longrightarrow \qquad \mathrm{Pol}_+(A, P_\Lambda)\mathrm{Pol}_-(A, P_\Lambda) = 0$$

This implies two possible solutions

$$A_+(P_\Lambda) \qquad \text{or} \qquad A_-(P_\Lambda)$$

Observation: $A_-(P_\Lambda)$ is the horizon area of the BPS black holes.



$$E|_{A_-} = 0$$

(i) Extremal black holes (BPS) with $A = A_-(P_\Lambda)$ and $\sum_\Lambda P_\Lambda = 0$ are stable.
(ii) Extremal black holes (non-BPS) with $A = A_+(P_\Lambda)$ and $\sum_\Lambda P_\Lambda = 0$ are unstable.

Writing in terms of invariants

▷ There exist extremal non-BPS black holes.

▷ We can distinguish them by computing invariants of $\mathrm{SL}(2,\mathbb{R})^3 \subset \mathrm{Sp}(8,\mathbb{R})$

In terms of these invariant

$$I_4 = \mathcal{I}(\Gamma,\Gamma,\Gamma,\Gamma) = 4\prod_\Lambda P_\Lambda$$

$$A^2_-(P_\Lambda) = \frac{3I_2}{2I_0} + \sqrt{\left(\frac{3I_2}{2I_0}\right)^2 - \frac{I_4}{4I_0}}$$

$$A^2_+(P_\Lambda) = \frac{3I_2}{2I_0} + \sqrt{\left(\frac{3I_2}{2I_0}\right)^2 + \frac{I_4}{12I_0}}$$

$I_4 < 0$

$I_4 > 0$

Perturbative stability and BF bound

In [Donos, Gauntlett, Pantelidou 1112.4195] were studied perturbations of the STU model on $\mathrm{AdS}_2 \times \mathbb{R}^2$ with constant equal scalars $\phi_{\mathcal{H}}$

$$\left.\begin{aligned} &\delta A^1 = \delta A^2 = \delta\phi_1 = 0\,, \\ &\delta\phi_2 = -\delta\phi_3 = \phi(t,\rho)\cos(\mathbf{k}x) \\ &\delta A^3 = -\delta A^4 = a(t,\rho)\sin(\mathbf{k}x)\mathrm{d}y\,. \end{aligned}\right\} \implies (\Box_{\mathrm{AdS}_2}\mathbb{1} - L^2\mathbb{M}^2)\begin{pmatrix}\phi\\ a\end{pmatrix} = 0$$

The eigenvalues of the mass matrix $\mathbb{M}^2$ are

$$m_{\pm}^2 = 3e^{\phi_{\mathcal{H}}} + \mathbf{k}^2 + e^{-\phi_{\mathcal{H}}} \pm e^{\phi_{\mathcal{H}}}\sqrt{8\mathbf{k}^2 e^{-\phi_{\mathcal{H}}}(2+e^{-2\phi_{\mathcal{H}}}) + (3+e^{-2\phi_{\mathcal{H}}})^2}\,.$$

m_-^2 has a minimum at $\mathbf{k}_{\min}$ giving: $\quad L^2 m_-^2|_{\min} = -\dfrac{(5+3e^{-2\phi_{\mathcal{H}}})}{48(2+3e^{-2\phi_{\mathcal{H}}}+e^{-4\phi_{\mathcal{H}}})}$

Perturbative stability and BF bound

In our analysis a simple expression for the value of the scalars at the horizon can be obtained

$$\phi_i(r_+) = -\log \frac{m^2 A}{(P_i^2 + A^2 L^{-2})(P_4^2 + A^2 L^{-2})} \,.$$

In the extremal configurations with $P_1 = P_2 = P_3$, we can apply the former results.

$\triangleright$ The BF bound is violated when $e^{-\phi_{\mathcal{H}}} < (-1 + \frac{2}{\sqrt{3}})^{1/4}$

$$\text{extremal BPS:} \qquad e^{-\phi_{\mathcal{H}}}|_{\text{BPS}} = (-1 + \frac{2}{\sqrt{3}})^{1/4} \,,$$

$$\text{extremal non-BPS:} \qquad e^{-\phi_{\mathcal{H}}}|_{\text{non-BPS}} = 1 \quad > (-1 + \frac{2}{\sqrt{3}})^{1/4}$$

Final remarks

Summary

4-charges electric black hole with arbitrary horizon curvature k

▷ The BPS limit leads to naked singularties $k = 1, 0$.

▷ Extremal and almost extremal planar black holes are unstable.

4-charges magnetic black holes with arbitrary horizon curvature

▷ There are BPS geometries that are well defined for $k = 1, -1, 0$

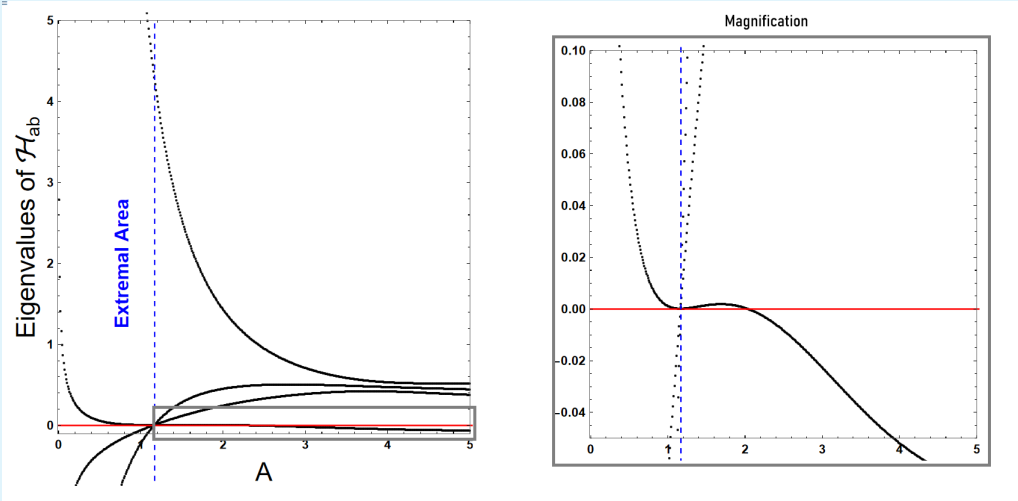
[Duff,Liu '98]∼[Cacciatori,Klemm 2010]

▷ There exist a suitable definition of thermodynamic energy that indicate stability for the BPS black holes.

▷ Extremal black holes do not imply BPS black holes.

Going beyond $T = 0$?

In the planar magnetic case with $\sum_\Lambda P_\Lambda = 0$



What can be done next?

- Explore if there is any insight for different topology at the horizon.
- What causes the instability?
- Study supersymmetry conditions for dyonic black holes, namely with axions [Compere,Chow 2013].
- What are the consequences of the instabilities in the dual theory?
- Extend to $D = 5$ STU model.

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Thanks! ☺